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Study On Feebly Lambda – Functions

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Abstract

in this research we study a several characterization of feebly λ-functions and investigate the relationship between such functions .

Keywords: feebly open set , λ-open set , feebly continuous function , feebly open mapping ,feebly closed mapping , feebly λ-continuous function , feebly λ-open mapping , feebly λ-closed mapping , perfectly continuous function , feebly λ-perfectly continuous function .

1. Introduction

The notion of feebly open set introduced by S. N. Maheshwari and P. C. Jain [1982] , after that some mathematician uses this definition in a topological space (X,T) , Dalal ibraheem in [2012] study feebly continuous and proved several results in her paper . Some Results of Feebly Open and Feebly Closed Mappings in introduced by dalal [2009] . Dalal ibraheem[2007] define the feebly generalized closed set also sina greenwood and ivan L. reilly [1986] introduced the feebly closed mappings with some result on It .S . Pious Missier and E. Sucila [2013] introduced the perfectly continuous function. Yiezi Al-talkany [2007] in this previous research we are define the λ-open set in bitopological space after that , H. shaheed and S. Kadham [2006] introduced the λ- continuous function in bitopological space. Now in this paper we define some feebly function by using the λ-open set and study some theorems.

2. 2-Preliminaries:

A subset A of a topological space X is said to be feebly open [S. N. Maheshwari and P.C. Jain 1982] if there exists an open set U such that $U \subseteq A \subseteq scl(U)$, Jankovic D. S. , Reidly I .L [1985] proved that the complement of feebly open set is feebly closed set

For a subset A of a space X the closure and interior of A with respect to a topological space T are denoted by cl(A) and int(A). Some basic theorems and definitions we needed in this paper we give it now:

2-1 Definition [Dalal Ibraheem 2007]

A function F:X→ Y is called feebly closed function if the image of each closed set in X is feebly closed set in Y.

2-2. Definition [Dalal Ibraheem 2009]

A function F: X → Y is called feebly open function if the image of each open set in X is feebly open set in Y .

2-3. Theorem [Dalal Ibraheem 2007]

Every open mapping is feebly open mapping.

2-4 Definition [H. Shaheed and S. Kadham2006]: a function f:(X,T,T^{α}) →(Y,V ,V^{α}) is called λ-continuous function if the inverse image of each open set in X is λ-open set in Y.

2-5 Definition [S. N. Maheshwari and P.C.Jain1982]: a function f:(X,T)→(Y,V) is said to be feebly continuous if the inverse image of each open set in Y is feebly open set in X .

2-6 Definition [yiezi AL-talkany 2007] : let (X,T,T^{α}) be a bitopological space a subset A in X is said to be λ-open set if there exist U$\in T^{\alpha}$ such that A$\subset U$ and A$\subseteq int_T$(U)

2-6 Remark: [Dalal Ibraheem 2009]

1- Every open set is feebly open set

2- Every closed set is feebly closed set

2-7 **Theorem: [Yiezi AL-Talkany2007]** every open set is λ-open set

2-8 **Theorem [H. Shaheed and S. Kadham 2006]** every continuous function is λ-continuous

2-9 Theorem [S. N.Maheshwari and P.C.Jain 1982] every continuous mapping is feebly continuous mapping.

Dalal Ibraheem in her research proof the following theorems:

2-10 Theorem:[Dalal Ibraheem 2009] every closed mapping is feebly closed mapping .

2-11 Theorem: [Dalal Ibraheem 2009]: every open mapping is feebly open mapping.

2-12 Theorem: [Dalal Ibraheem 2009] the composition of two closed function is feebly closed function .

all the above theorems are not exist in our research.

3 Feebly λ-continuous function

Definition : a function $f:(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ is said to be feebly λ-continuous iff the inverse image for every λ-open set in Y is feebly open set in X .

3-1 Theorem: [Saad Naji AL-Azawi , Jamhour Mahmoud AL-obaidi, Aco Saied2008]

Every continuous function is feebly continuous function

3-2 Theorem : if the function $f:(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ is feebly λ-continuous then

$f:(X,T) \rightarrow (Y,V)$ is feebly continuous .

Proof: let H is open set in Y , by remark (1-5) H is λ-open set , since f is feebly λ-continuous then $f^{-1}(H)$is feebly open set and then f is feebly continuous.

3-3 Theorem:

Let $f:(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ is feebly λ-continuous and $g:(Y,V,V^{\alpha}) \rightarrow (Z,W,W^{\alpha})$ is λ-continues then gof is feebly λ-continuous

Proof: let A is open set in Z , since g is λ-continuous then $g^{-1}(A)$ is λ-open set in Y and since f is feebly λ-continuous then

$f^{-1}(g^{-1}(A))=(gof)^{-1}(A)$ is feebly open set in X .

3-4 Theorem : Let $f:(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ is feebly λ- continuous and $g:(Y,V,V^{\alpha}) \rightarrow (Z,W,W^{\alpha})$ is λ-continuous then gof is feebly λ-continuous

Proof : exist by definitions

3-6 Theorem : Let $f:(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ be a map then the following are equivalent :

1- f is feebly λ-continuous
2- The inverse image of each λ-closed set in Y is feebly closed set in X
3- $Cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for each A in Y
4- $f(cl(A)) \subseteq cl(f(A))$ for each A in X
5- $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for each B in Y

Proof: (1)$\Rightarrow$ (2) obvious by definition

(2)$\Rightarrow$(3) let A is subset of Y , then cl(A) is closed set in Y and then it is λ-closed set in Y , by (2) $f^{-1}(cl(A))$ is feebly closed set in X .

Since $f^{-1}(A) \subseteq f^{-1}(cl(A))$ then $cl(f^{-1}(A)) \subseteq cl(f^{-1}(cl(A))= f^{-1}(cl(A)$.

(3)$\Rightarrow$(4)let A is closed set in X , then by (3) we get $cl(A) \subseteq cl(f^{-1}(f(A))) \subseteq f^{-1}(cl(f(A))$ then $f(cl(A)) \subseteq cl(f(A))$.

(4)$\Rightarrow$(5)let B is any sub set of Y , by (4) $f(cl(X-f^{-1}(B)) \subseteq cl(f(X-f^{-1}(B))$and then

$f(X-int(f^{-1}(B)) \subset cl(Y-B)=Y-int(B)$ then we get that $X-int(f^{-1}(B)) \subset -f^{-1}(Y-int(B))$ and then

$f^{-1}(int(B)) \subset int(f^{-1}(B))$.

(5)$\Rightarrow$(1) let A is λ-open set in Y , then by (5) $f^{-1}(int(A)) \subset int(f^{-1}(A))$ and then

$f^{-1}((A)) \subset int(f^{-1}(A))$, from that we get $f^{-1}(A)$ is feebly open set in X.

3-7 Example:

X={1,2,3,4} , T = {, X , {1},{1,2,3}} and Y={a,b,c},V={Y,{a},{a,b}} , then λ-open set={X,{a},{b},{a,b}} and f:X→Y defined by f(1)=f(2)=a , f(3)= f(4)= b. then f is feebly continuous but not feebly λ-continuous since f^{-1}({b})={3,4} which is not feebly open set in X .

4 -feebly λ-open function and feebly λ-closed function

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4-1. Definition :a function $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is said to be feebly λ-open function if f(G)is feebly open set in Y foe every λ-open set G in X .

4-2. Definition :a function $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is said to be feebly λ-closed if f(G) is feebly closed set in Y for every

λ-closed set G in X

4-3. Theorem : let $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is feebly λ-open and bijective function then f is feebly λ-closed function .

Proof: let H is λ-closed set in X then X-H is λ-open set , since f is bijective then

f(X-H)=Y-f(H) is feebly open set in Y and then f(H) is feebly closed set inY .

4-4 . Theorem : let $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ and $g: (Y,V,V^{\alpha}) \to (Z,W,W^{\alpha})$ are two function such that Gof is λ-open function and g is feebly λ-continuous injective function then f is feebly λ-open function.

Proof: let A is λ-open set in X , then(gof)(A) is λ-open function , since g is feebly λ-continuous ,then g^{-1}(gof)(A)=f(A) is feebly open set in Y , and then f is feebly λ-open function.

4-5. Theorem : let $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ and $g: ((Y,V,V^{\alpha}) \to (Z,W,W^{\alpha})$ are two functions such that Gof is feebly -open function and f is feebly λ-continuous surjective function then g is feebly λ -open function .

Proof: let B is λ-open set inY , since f is feebly λ-continuous then f^{-1}(B) is feebly open set in X, and since gof is feebly open function ,then (gof)(f^{-1}(B))=g(B) is feebly open set in Z.

4-6. Theorem : let $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ and $g: ((Y,V,V^{\alpha}) \to (Z,W,W^{\alpha})$ are two function such that Gof is λ-closed function and g is feebly λ-continuous injective function then f is feebly λ-closed function .

Proof: let H is λ-closed set in X , then (gof)(H) is λ-closed set in Z, since g is feebly λ-continuous then g^{-1}(gof)(H)=f(H) is feebly closed set in Y , f is feebly λ-closed map.

4-7. Theorem : let $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ and $g: ((Y,V,V^{\alpha}) \to (Z,W,W^{\alpha})$ are two function such that f is λ-closed function and g is feebly λ-closed function then go f is feebly λ-closed function .

Proof: let H is λ-closed set in X , then f(H) is λ-closed set in Y and then g(f(H) =gof(H) is feebly closed set in Z , then gof is feebly λ-closed function.

5- Feebly λ-perfectly continuous function

5-1. Definition [S.Pious Missier and E. Sucila ,2013]:

A mapping f:(X ,T)→(Y,V) is said to be perfectly continuous if the inverse image of each open set in Y is both open and closed in X

5-2. Definition :A function $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is said to be feebly λ-perfectly continuous function if the inverse image of each λ-open set in Y is feebly open set and feebly closed set in X .

5-3. Theorem: every feebly λ-perfectly continuous function is feebly continuous function.

Proof: let A is open set in Y , and then it is λ-open set since f is feebly λ-perfectly continuous , then f^{-1}(A) is feebly open set in X and then f is feebly continuous .

5-4. Theorem : Every feebly λ-perfectly continuous function is feebly λ-continuous function.

Proof: exist by definitions

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